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Damping lateral vibrations in rotary machinery using motor speed modulation

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ABSTRACT

This paper outlines a new principle for damping lateral vibrations of rotary systems. According to this principle, no changes in the visco-elastic properties of the system to be damped are required. The method is based on the generation of a harmonic additive to the constant speed of rotation that provides significant damping of lateral vibrations at critical speeds of rotation. This concept is validated analytically using the method of averaging and additionally with the help of direct numerical integration. The solution is shown to represent Fourier series containing Bessel functions. Consequently, proper choice of the parameters of the additional harmonic component ensuring that the Bessel functions have minimum values results from a minimization of the solution itself. Thus, the analytical solution and numerical results prove this concept by showing an essential decrease of the amplitudes of lateral vibrations of the damped system compared with those of the undamped system. The physical explanation of this effect is presented.

1. Introduction

Experimental studies and exploitation of rotating machinery (turbines, centrifugal pumps, gears, etc.) show that there exist ranges of rotation speeds for which vibration can reach undesirably high values. This situation can happen both during the startup process, when the system has to pass the critical speeds, and during the shutdown process. Sometimes, the designed operating speed may happen to be close to a critical speed. Finally, a shift of natural frequencies towards the operating speed can occur. Such a change in oscillatory properties can occur during the operation of such machinery due to numerous reasons, such as wear of bearings, change of the mass, and imbalance of impellers due to sedimentation of deposits [1,2].

There exist a large number of papers and patents related to different vibration damping techniques. These publications can be subdivided into several large groups using the damping technique as a classification criterion. The most common method is the so-called structural vibration damping [3–12], which is generally based on the application of structural elements of various complexities. In particular, elastomers [13–15], special coatings [17,18], or even shape memory alloys [19] are commonly used. Rotating machinery may also have self-balancing devices [20]. Vibrations may be damped using journal bearings that employ either liquid [21–23] or a compressed air [24]. The application of an electromagnetic field allows for the dampening of vibrations using rheological fluid [25,26], piezo-electric actuators, magnetostrictors, magnetic bearings, etc. [27–38]. Systems of different complexities can be designed for vibration damping control [39].

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Here, we study the damping of lateral vibrations of a rotary system by control of the rev/min; i.e., controlling the motor speed [42]. This technique is totally different from all of the aforementioned vibration damping methods and requires no changes of visco-elastic properties.

2. The model

The system under consideration is represented by a motor coupled to an unbalanced disk placed in the center of a weightless and torsionless shaft (see Fig. 1). Here, e is the disk eccentricity, W is the geometric center of the disk, point W designates the disk geometrical center, point S designates its center of gravity, and point O designates the axis of the unperturbed shaft. When rotor speed (frequency of rotation) approaches a resonance zone, a harmonic component is added to the constant torque. Such a harmonic additive may also be added during the operating mode (behind the resonance zone) when the control system indicates an undesirably high amplitude of lateral vibration. When the damping system is turned on, the constant motor speed undergoes harmonic modulation. The governing equations have the form

$$\ddot{x} + \frac{\varepsilon\omega_0}{c}\dot{x} + x + \frac{k\omega_0}{c}(\dot{x} + \dot{\varphi}y) = e\cos\varphi,$$
$$\ddot{y} + \frac{\varepsilon\omega_0}{c}\dot{y} + y + \frac{k\omega_0}{c}(\dot{y} - \dot{\varphi}x) = e\sin\varphi,$$
$$\dot{\varphi} = \Omega + An\Omega\cos(n\Omega\tau + \psi_0). \quad (1)$$

Here, x and y are the coordinates of the disk's center of gravity in the space-fixed coordinate system perpendicular to the unperturbed shaft axis, whose origin is located at the shaft; ε and k are the coefficients of external and internal damping, respectively; c is the lateral stiffness of the shaft; φ is the angle of rotation, ω_0 is the first natural frequency of the rotor; ω is the speed of rotation; $\Omega=\omega/\omega_0$ is the non-dimensional speed of rotation; A and ψ_0 are the amplitude and the phase shift of the harmonic additive; n is an integer. Variables, parameters and time are dimensionless in system (1). Differentiation is done over a dimensionless time $\omega_0 t=\tau$.

We suppose that the coefficients of external and internal damping as well as the rotor eccentricity are small, i.e.

$$c/m = \omega_0^2,\ \varepsilon\omega_0/c = \mu h,\ k\omega_0/c = \mu h_1,\ e = \mu v,\ \Omega^2 - 1 = \mu\Delta,$$

where $\mu \ll 1$ is a small parameter. In this case, system (1) has the form

$$\dot{x} = x_1,\ \dot{x}_1 = -\Omega^2 x + \mu F_1,\ \dot{y} = y_1,\ \dot{y}_1 = -\Omega^2 y + \mu F_2,\ \dot{\psi} = \Omega. \quad (2)$$

Here, $F_1 = v\cos\varphi - (h+h_1)x_1 - h_1\dot{\varphi}y + \Delta x$, $F_2 = v\sin\varphi - (h+h_1)y_1 + h_1\dot{\varphi}x + \Delta y$, $\varphi=\psi+A\sin(n\psi+\psi_0)$.

Our goal is to answer the question: would it be possible to choose the parameters of modulation A, n, ψ_0 in such a way that the amplitudes of lateral vibrations would be minimized, and how could the choice of these parameters be optimized?

Systems (1) and (2) are the systems of two non-autonomous linear oscillators. Each of these oscillators represents a resonance filter of the frequency Ω, which corresponds to the first harmonic in the spectrum of external disturbance

$$\cos\varphi = \cos(\psi + A\sin(n\psi + \psi_0)),\ \ \sin\varphi = \sin(\psi + A\sin(n\psi + \psi_0)),\ \ \psi = \Omega\tau.$$

These functions may be decomposed into the Fourier series with coefficients $J_k(A)$ representing Bessel functions of the first kind of integer argument. Naturally, because the model under consideration is linear, then solutions can also be represented as Fourier series. If the first harmonic that has the highest amplitude would be damped by a proper choice of the parameters of modulation, then the amplitudes of the remaining harmonics will have values of the smaller order of $\sim\mu \ll 1$. This characteristic, explained by the filtering properties of the oscillators, means that the maximum radial displacements x_{max}, y_{max} will have the same order $\sim\mu$ (note that $x(\tau)$, $y(\tau)$ are the mutifrequency functions). Thus, our objective is to provide damping of the first harmonic.

To solve this problem, one has to first search for the solution of the system (2) in a form of the Fourier series with undetermined coefficients, and second, determine these coefficients and further minimize the amplitude of the first harmonic. At the same time, the stability of the solution must be ensured. The problem is customarily approached using the

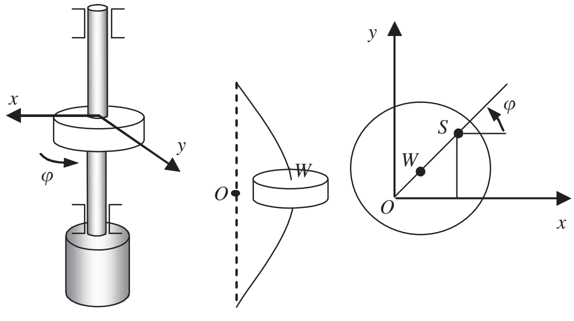

Fig. 1. The model. *Left*: shaft with imbalanced disk; middle: shaft deflection; *right*: disk plane (view from above) showing coordinate plane and eccentricity.

method of averaging [40,41], which can be briefly explained as follows. First, the original system, which is defined in a cylindrical phase space, is reduced to a system in a standard form with a fast spinning phase [40] using changes of the variables and the introduction of a small parameter. Second, the obtained system is averaged over the fast spinning phase. Third, the existence of a globally asymptotically stable invariant manifold in the phase space of the averaged system is to be proved. This step helps to replace the consideration of the averaged system with consideration of a simpler system, whose phase space is placed at the invariant manifold. This system has a smaller dimension compared with the averaged one but at the same time contains all the averaged system's qualitative properties. The method of averaging is explained in detail in [40,41].

Using the change of the variables of the form

$$x = u_1 \sin\psi + v_1 \cos\psi, \quad x_1 = (u_1 \cos\psi - v_1 \sin\psi)\Omega,$$
$$y = u_2 \sin\psi + v_2 \cos\psi, \quad y_1 = (u_2 \cos\psi - v_2 \sin\psi)\Omega,$$

we reduce system (2) to the standard form with a fast spinning phase [40,41]

$$\dot{u}_1 = \mu F_1 \cos\psi,\ \dot{v}_1 = -\mu F_1 \sin\psi,\ \dot{u}_2 = \mu F_2 \cos\psi,\ \dot{v}_2 = -\mu F_2 \sin\psi,\ \dot{\psi} = \Omega. \tag{3}$$

The method of averaging significantly simplifies further study, namely by replacing the study of periodic motions in the original system (3) by the study of equilibriums of the averaged system. Prior to the averaging procedure [44], let us note that the values of the variables

$$\langle \dot{\varphi} y \sin\psi \rangle_\psi, \langle \dot{\varphi} y \cos\psi \rangle_\psi, \langle \dot{\varphi} x \sin\psi \rangle_\psi, \langle \dot{\varphi} x \cos\psi \rangle_\psi,$$

$$\langle \cos(\psi + A\sin(n\psi+\psi_0))\sin\psi \rangle_\psi, \langle \cos(\psi + A\sin(n\psi+\psi_0))\cos\psi \rangle_\psi,$$
$$\langle \sin\psi(\psi + A\sin(n\psi+\psi_0))\sin\psi \rangle_\psi, \langle \sin(\psi + A\sin(n\psi+\psi_0))\cos\psi \rangle_\psi,$$

strongly depend on the parameter n. Namely, the right-hand sides of the equations of the averaged system are different for different n. For this reason, we have to consider three qualitatively different cases: (1) n is either any integer number from the interval $n>2$, any fractional number from the interval $0<n<1$ or any irrational one; (2) n=1; (3) n=2. Let us now consider each case in more detail and illustrate them with the results of numerical modelling.

Case 1: n is either any integer number from the interval $n>2$, any fractional number of the interval $0<n<1$, or any irrational one. Having averaged system (3) over the fast-spinning phase ψ and transforming the time: $\mu\tau=\tau_{new}$, we obtain equations in the first approximation with respect to the small parameter. These equations have the form

$$\begin{aligned}\dot{u}_1 &= -\frac{h+h_1}{2}u_1 - \frac{h_1}{2}v_2 + \frac{\Delta}{2}v_1 + \frac{1}{2}vI_0(A),\\ \dot{v}_1 &= -\frac{h+h_1}{2}v_1 + \frac{h_1}{2}u_2 - \frac{\Delta}{2}u_1,\\ \dot{u}_2 &= -\frac{h+h_1}{2}u_2 + \frac{h_1}{2}v_1 + \frac{\Delta}{2}v_2,\\ \dot{v}_2 &= -\frac{h+h_1}{2}v_2 - \frac{h_1}{2}u_1 - \frac{\Delta}{2}u_2 - \frac{1}{2}vI_0(A).\end{aligned} \tag{4}$$

Here $I_0(A)$ is the Bessel function of the first kind [45]. In this case, equations are independent of a phase shift ψ_0.

Value $A^* = \sqrt{u_{10}^2 + u_{20}^2 + v_{10}^2 + v_{20}^2}$, where u_{10}, u_{20}, v_{10}, v_{20} are the coordinates of the equilibrium of linear system (4), represents the amplitude of the first harmonic. Our goal is to minimize this value.

Note that system (4) has a stable invariant manifold M={u_1=$-v_2$, u_2=v_1} [43].

Consider now system at the manifold M={u_1=$-v_2$=u, u_2=v_1=v}:

$$\dot{u} = -\frac{h}{2}u + \frac{\Delta}{2}v + \frac{v}{2}I_0(A), \quad \dot{v} = -\frac{h}{2}v - \frac{\Delta}{2}u. \tag{5}$$

Values of the coordinates of equilibrium (u_0, v_0) of system (5) are proportional to $I_0(A)$. The amplitude of the first harmonic is minimal for minimal values of $|I_0(A)|$ from the interval of allowed values of the amplitude of modulation A. The amplitude of the first harmonic is equal to zero (full damping) for all A, for which $I_0(A)$=0. The Bessel function $I_0(A)$ has an infinite number of zeroes. In particular, the first zero corresponds to A=2.4 (minimal value of A). Having substituted A=2.4 into system (5), we obtain a stable equilibrium u=0, v=0.

Thus, having chosen A=2.4, any integer $n>2$ or any irrational n, and any value of the phase shift ψ_0 (for instance, ψ_0=0), we obtain the full damping of the first harmonic of rotor lateral vibration. In this case, the amplitude of lateral vibration comes to be of the order $\sim\mu\ll 1$.

Numerical study: Because internal damping does not affect the amplitude of lateral vibrations, instead of system (1), we have studied an equivalent system of the form

$$m\ddot{x} + \varepsilon\dot{x} + cx = ce\cos\varphi, \quad m\ddot{y} + \varepsilon\dot{y} + cy = ce\sin\varphi, \quad \dot{\varphi} = \Omega + An\Omega\cos(n\Omega t).$$

Hereinafter, the following dimensionless parameters are used:

$$m = 1, \varepsilon = 0.1, c = 25, e = 1.$$

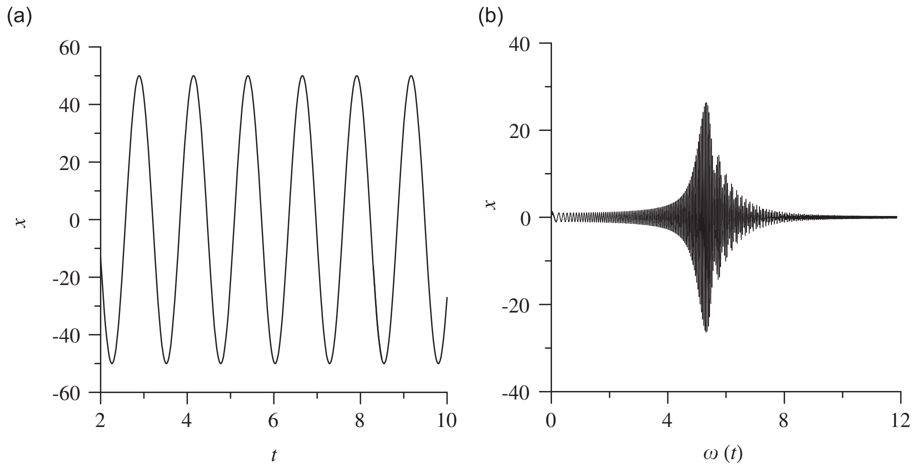

Fig. 2. (a) time-history of solution $x=x^*(t)$ of system (1) for the undamped case, (b) lateral displacement $x(\omega(t))$ during the startup process and passage through the first critical speed.

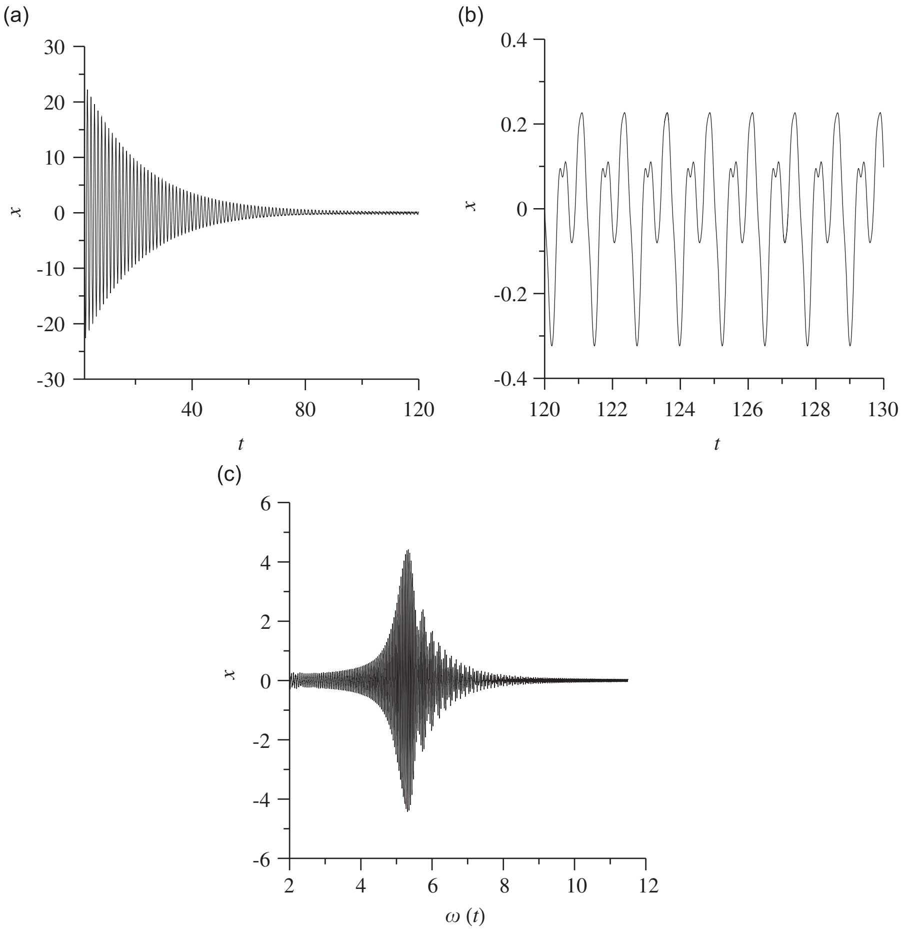

Fig. 3. The case of $n=3$, $A=2.4$: (a) transient process of damping right after the modulation is switched on, (b) solution $x=x^*(t)$ versus time as a result of damping at the first natural frequency, (c) lateral displacement $x(\omega(t))$ during the startup process and passage through the resonance zone, when the modulation is continuously turned on.

Fig. 2a shows a time-history of solution $x=x^*(t)$ of system (1) for the undamped case $n=0$. Fig. 2b shows the lateral displacement $x(\omega(t))$ during the startup process and passage through the resonance zone.

Now, let us study what happens if the damping system is turned on, i.e., in the case when the motor speed modulation is on. Generally, modulation can either be switched on at the certain moment of time when the system approaches the

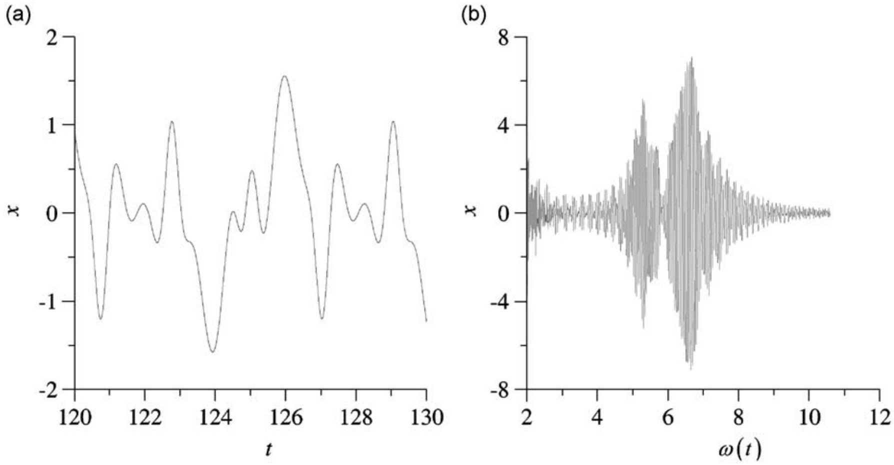

Fig. 4. The case of n=0.6, A=2.4: (a) solution $x=x^*(t)$ versus time as a result of damping at the first natural frequency, (b) lateral displacement $x(\omega(t))$ during the startup process and passage through the resonance zone, when the modulation is continuously turned on.

resonance zone, or can be turned on continuously right after system has started up. Let us consider both these cases. Fig. 3a shows the transient process right after the modulation is turned on at the 20th second (n=3, A=2.4); one can see that the vibration starts to decay immediately and later reach the very small values shown in Figs. 3b and 4a for the cases n=3, A=2.4 and n=0.6, A=2.4, respectively. Consider now the case when the modulation is turned on continuously. Lateral displacement of the shaft $x(\omega(t))$ during the startup process and passage through the resonance zone is shown in Figs. 3c and 4b for the cases n=3, A=2.4 and n=0.6, A=2.4, respectively.

For n=3, A=2.4, when the modulation is turned on near the first natural frequency, the damping ratio (ratio between the maximal amplitudes of lateral vibration of damped and undamped systems) equals 217.4. For the continuously turned-on damping system, when the system passes through the resonance zone, the damping ratio equals 7.7. The difference between these coefficients is a result of the high Q-factor of the oscillatory system and due to excitation of self-vibrations.

For n=0.6, A=2.4, when modulation is turned on near the first natural frequency, the coefficient of damping equals 33.3. For the continuously turned-on modulation, when the system passes through the resonance zone, the coefficient of damping equals 7.

Case 2: n=1 (modulation at the rotor speed). In this case, the averaged system has the form

$$\dot{u}_1 = -\frac{h+h_1}{2}u_1 - \frac{h_1}{2}v_2 + \frac{\Delta}{2}v_1 + \frac{v}{2}(I_0 + I_2\cos 2\psi_0), \quad \dot{v}_1 = -\frac{h+h_1}{2}v_1 + \frac{h_1}{2}u_2 - \frac{\Delta}{2}u_1 + \frac{v}{2}I_2\sin 2\psi_0,$$
$$\dot{u}_2 = -\frac{h+h_1}{2}u_2 + \frac{h_1}{2}v_1 + \frac{\Delta}{2}v_2 - \frac{v}{2}I_2\sin 2\psi_0, \quad \dot{v}_2 = -\frac{h+h_1}{2}v_2 - \frac{h_1}{2}u_1 - \frac{\Delta}{2}u_2 - \frac{v}{2}(I_0 - I_2\cos 2\psi_0). \tag{6}$$

The equilibrium of system (6) is stable. This equilibrium has zero coordinates independently of the phase ψ_0 if $I_0(A)=I_2(A)=0$ (normally, initial phase is hard to control). However, this system is inconsistent. In contrast to the previous case, for n=1, there are no values of the parameter A for which the damping of the first harmonic would be full. Nevertheless, there exists a value of amplitude of the modulator for which the amplitude of lateral vibration would be minimal.

Numerical study: Fig. 5 shows the lateral displacement of the shaft $x(\omega(t))$ during the startup process and passage through the resonance zone when the modulation is continuously turned on for the case of n=1, A=5.1.

When modulation is turned on near the first natural frequency, the damping ratio equals 6.4. For the continuously turned on damping system, when the system passes through the resonance zone, the damping ratio equals 11.1.

Case 3: n=2 (modulation at the doubled rotor speed). In this case, averaged system has the form

$$\dot{u}_1 = -\frac{h+h_1}{2}u_1 - \frac{h_1}{2}v_2 + \frac{\Delta}{2}v_1 - \frac{h_1A}{2}(-u_2\sin\psi_0 + v_2\cos\psi_0) + \frac{v}{2}(I_0 - I_1\cos\psi_0),$$
$$\dot{v}_1 = -\frac{h+h_1}{2}v_1 + \frac{h_1}{2}u_2 - \frac{\Delta}{2}u_1 + \frac{h_1A}{2}(-u_2\cos\psi_0 - v_2\sin\psi_0) - \frac{v}{2}I_1\sin\psi_0,$$
$$\dot{u}_2 = -\frac{h+h_1}{2}u_2 + \frac{h_1}{2}v_1 + \frac{\Delta}{2}v_2 + \frac{h_1A}{2}(-u_1\sin\psi_0 + v_1\cos\psi_0) + \frac{v}{2}I_1\sin\psi_0,$$
$$\dot{v}_2 = -\frac{h+h_1}{2}v_2 - \frac{h_1}{2}u_1 - \frac{\Delta}{2}u_2 - \frac{h_1A}{2}(-u_1\cos\psi_0 - v_1\sin\psi_0) - \frac{v}{2}(I_0 + I_1\cos\psi_0). \tag{7}$$

Numerical study: Fig. 6 shows lateral displacement of the shaft $x(\omega(t))$ during the startup process and passage through the resonance zone, when the modulation is continuously turned on (n=2, A=5.5).

When modulation is turned on near the first natural frequency, the damping ratio equals 2.85. For the continuously turned on modulation, when system passes through the resonance zone, the damping ratio equals 33.3.

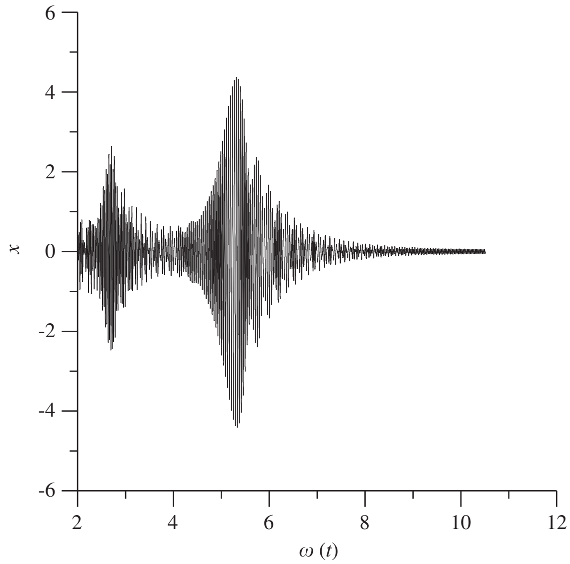

Fig. 5. Lateral displacement $x(\omega(t))$ during the startup process and passage through the resonance zone when the modulation is continuously turned on. n=1, A=5.1.

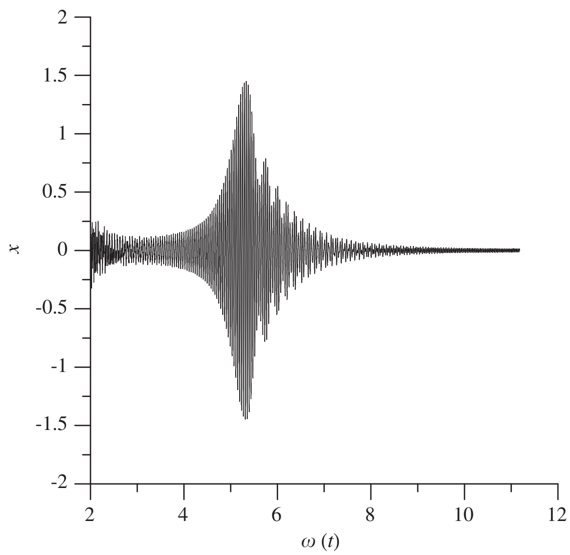

Fig. 6. Lateral displacement $x(\omega(t))$ during the startup process and passage through the resonance zone, when the modulation is continuously turned on. The case of n=2, A=5.5.

It is clear that for real systems, the right choice of the parameters of damping should take into account system inertia, the possibility of generation of undesired torsional vibrations, etc. Fig. 7 shows the maximal dimensionless lateral displacement normalized by a maximal displacement at the first resonant frequency versus n for A=2.4 (Fig. 7a) and versus A for n=0.2 (Fig. 7b). This helps understand how the optimal pair of control parameters could be chosen.

We would like to note that the proposed process of vibration damping could be implemented as an integral part of automatic control system.

3. Conclusions

We propose a new technique for damping the lateral vibrations of rotary machinery, based on the generation of an additional harmonic component to the constant speed of rotation of the form $An\Omega\cos(n\Omega\tau+\psi_0)$. It is desirable to select the smallest values A and n among the possible ones. Note that the number n can be not only an integer number but also a fractional or even irrational number. The motor plays a role of a control system. The harmonic component in rev/min may

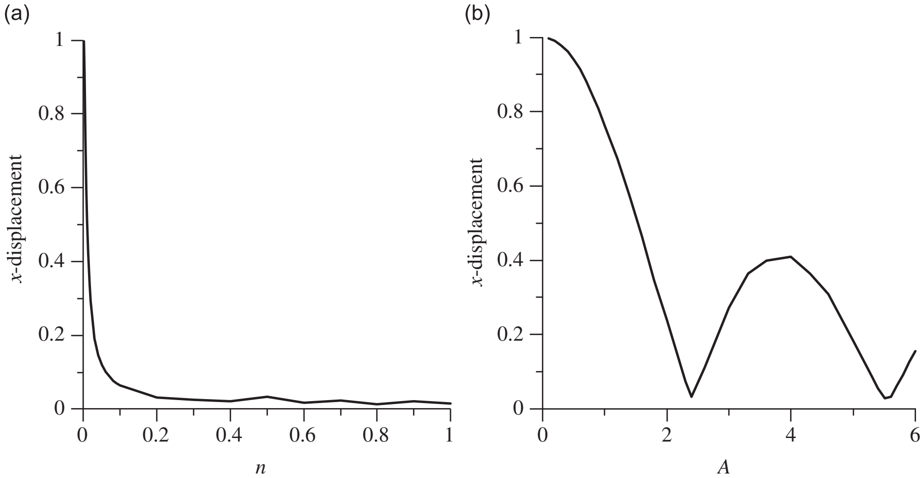

Fig. 7. Dimensionless maximal lateral displacement normalized by a maximal displacement at the first resonance frequency $\omega = \sqrt{c/m}$): (a) versus n for A=2.4, (b) versus A for n=0.2.

be realized through a harmonic additive to the electric current. For each rotary system, the best set of A and n may be determined through mathematical models and experimental study. Generally speaking, there exist an infinite set of pairs for A and n, and for the each one, the degree of damping will be different. An active vibration damping control system can be made based on this concept. Experimental verification of the proposed technique is required.

Acknowledgements

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